Module 9

DC Machines

Lesson 36

Principle of Operation of D.C Machines

Contents

36.1 Introduction

In the first section of the lesson we consider an example using a single conductor to behave as a generator or as a motor. Although the motion of the conductor is rectilinear, the example brings out several useful facts which are true for rotating machines as well.

The following sections begin by giving some important information which are true for *all most all kinds* of rotating electrical machines. It is first explained that two torques namely the driving torque and the load torque will exist during the operation of the machine both as generator and motor. After going through the lesson one will understand:

1. the meaning of loading of generator and motor.
2. that motoring & generating actions go side by side.
3. the dynamics involved while the operating point moves from one steady state condition to another.

36.2 Example of Single conductor Generator & Motor

1. **Generator Mode:** Consider a straight conductor of *active length* (the length which is under the influence of the magnetic field) l meter is placed over two friction less parallel rails as shown in the figure 36.1. The conductor is moving with a constant velocity v meter/second from left to right in the horizontal plane. In the presence of a vertical magnetic field directed from top to bottom of strength B Wb/m^2, a voltage $e = Blv$ will be induced across the ends of the moving conductor. The magnitude of the voltage will be constant and the polarity will be as shown in the figure 36.2. In other words the moving conductor has become a seat of emf and one can replace it by battery symbol with an emf value equal to Blv Volts.

   At no load i.e., (resistance in this case) is connected across the moving conductor, output current hence output power is zero. Input power to the generator should also be zero which can also be substantiated by the fact that no external force is necessary to move a mass with constant velocity over a frictionless surface. The generator is said to be under no load condition. Let us now examine what is going to happen if a resistance is connected across the source. Obviously the conductor starts delivering a current $i = \frac{e}{R}$ the moment resistance is connected. However we know that a current carrying conductor placed in a magnetic field experiences a force the direction of which is decided by the *left hand rule*. After applying this rule one can easily see that the direction of this electromagnetic force will be opposite to the direction of motion i.e., v. As told earlier that to move the conductor at constant velocity, no external force hence prime mover is not necessary. Under this situation let us assume that a load resistance R is connected across the conductor. Without doing any mathematics we can purely from physical reasoning can predict the outcome.

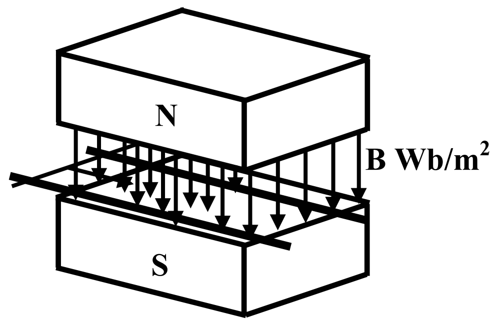


Figure 36.1: Elementary Generator

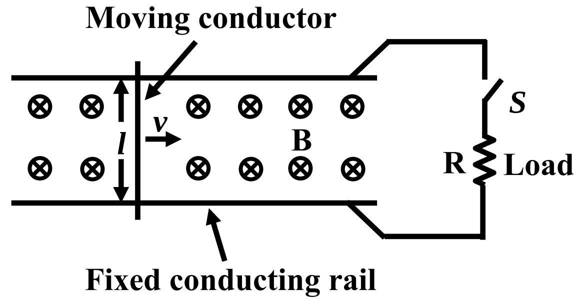


Figure 36.2: Top view of figure 36.1

The moment load is connected, the conductor starts experiencing a electromechanical force in the opposite direction of the motion. Naturally conductor starts decelerating and eventually comes to a stop. The amount of energy dissipated in the load must have come from the kinetic energy stored in the conductor.

Let us now Analyse the above phenomena mathematically. Suppose,

v_0 = linear velocity of the conductor in meter/sec under no load condition.

t = 0, is the instant when the load is switched on.

v = linear velocity of the conductor in meter/sec at any time t.

l = length of the conductor in meters.

m = mass of the conductor in Kg.

B = flux density in Wb/meter sq.

e = Blv, induced voltage at any time, t.

R = load resistance in Ω.

i = $\frac{e}{R}$, current in A at any time, t.

F_e = Bil, electromagnetic force in opposite direction of motion, at time, t. (36.1)

The dynamic equation of motion of the conductor can be written by using Newton's law of motion as follows:

$$m\frac{dv}{dt} = -F_e = -Bil = -Bl\left(\frac{Blv}{R}\right)$$

$$m\frac{dv}{dt} + \frac{B^2l^2}{R}v = 0$$

$$\frac{dv}{dt} + \frac{B^2l^2}{mR}v = 0 \text{ dividing both sides by } m. \qquad (36.2)$$

Solving this linear simple first order differential equation and applying the boundary condition that at $t = 0$, $v = v_0$ we get the expressions for velocity, emf and current as a function of time.

$$
\begin{aligned}
v &= v_0 e^{-\frac{B^2 l^2}{mR}t} \\
e &= B l v_0 e^{-\frac{B^2 l^2}{mR}t} \\
i &= \frac{B l v_0}{R} e^{-\frac{B^2 l^2}{mR}t}
\end{aligned}
\qquad (36.3)
$$

From the above we see that in absence of any external agency for motive power, the velocity and current decreases exponentially with a time constant $\tau = \frac{mR}{B^2 l^2}$ down to zero, as shown in the figure 36.3. We can easily calculate the amount of energy W_R dissipated in R and show the same to be equal to the initial kinetic energy $\left(\frac{1}{2} m v^2\right)$ by carrying out the following integration.

$$
W_R = \int_0^\infty i^2 R dt = \frac{1}{2} m v^2 \qquad (36.4)
$$

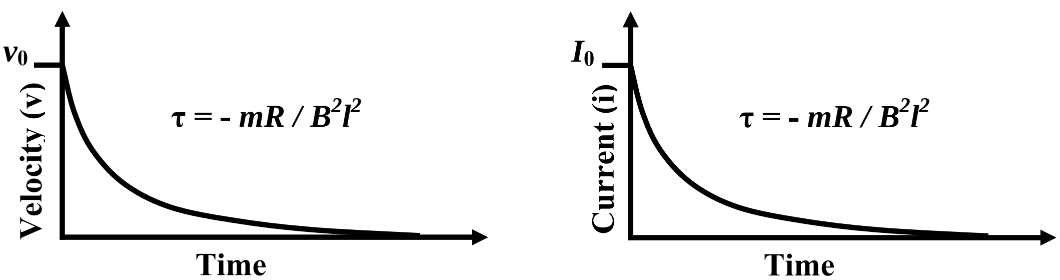


Figure 36.3: Velocity and Current variation

Thus for sustained generator operation some external agency (prime mover) must apply a force in the direction of motion to counter balance opposite electromagnetic motoring force. Under such situation, for a particular steady load current, the conductor can move with constant velocity maintaining sustained generator operation. Power delivered to the load comes from the external agency (prime mover) as it is doing work against the opposing electromagnetic force.

A Numerical Example

A single conductor generator of 2 m active length and zero resistance (as explained above) is found to deliver power to a 5 Ω load resistance. The generator is found to move with a constant velocity of 5m/sec under the influence of a magnetic flux density of 1.1 wb/m^2. Calculate the voltage impressed across the load and current supplied to it. Also calculate the force exerted by the mechanical external agency and the mechanical power supplied to it.

Solution

Here,

$$
\begin{aligned}
B &= 1.1 \text{ wb/m}^2 \\
l &= 2 \text{ m} \\
v &= 5 \text{ m/sec}
\end{aligned}
$$

$$
\begin{aligned}
\text{voltage generated } e &= Blv \\
&= 1.1 \times 2 \times 5 \\
\therefore e &= 11 \text{ V} \\
\text{voltage across the load} &= 11 \text{ V} \\
\text{current in the load and generator, } I &= 11/5 = 2.2 \text{ A} \\
\text{Power supplied to the load resistance, } P_R &= 2.2^2 \times 5\text{W} \\
\text{electromagnetic force developed in the generator, } F_e &= 4.84 \text{ N}
\end{aligned}
$$

It may be noted that $F_e = 4.84$ N acts in a direction opposite to the direction of motion. But as the conductor is moving with a constant velocity, the external agency must be applying a mechanical force of $F_m = 4.84$ N in the direction of motion. The mechanical power input to the generator is $P_m = F_m v = 4.84 \times 5 = 24.2$ W. So $P_R = P_m$ is not surprising as we have assumed that there is no power loss in the generator.

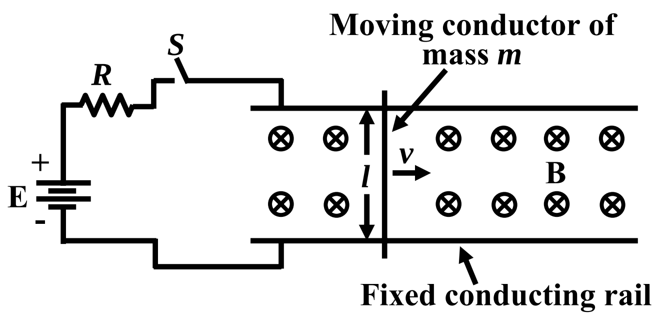


Figure 36.4: Top view during motor mode

2. **Motor Mode:** The same arrangement can be used to demonstrate motoring action as shown in figure 36.4
   Suppose the conductor is initially stationary, and battery of emf E is connected across it through a resistance R. Obviously the current right at the time of connecting the battery i.e., at $t = 0$, is $I = \frac{E}{R}$. As the current carrying conductor is placed in a magnetic field it will experience a force BIl and the conductor will start moving. However when the conductor starts moving a voltage is induced across the conductor the polarity of which will be such so as to oppose the inflow of current into the conductor. Therefore with time both the value of the current and the electromagnetic torque will decrease. To answer what will happen finally to the current and the speed of the conductor, we shall write the following electrodynamic equation and solve them.

t = 0, is the instant when the battery is switched on.

v = linear velocity of the conductor in meter/sec at any time t.

l = length of the conductor in meters.

m = mass of the conductor in Kg.

B = flux density in Wb/meter sq.

e = Blv, induced voltage at any time, t.

R	=	resistance connected in series with the battery Ω.
i	=	$\frac{E - e}{R}$, current in A at any time, t.
F_e	=	Bil, electromagnetic force causing motion, at time, t. (36.5)

The equations of motion in case of generator mode are as follows:

$$m\frac{dv}{dt} = F_e = Bil = Bl\left(\frac{E - Blv}{R}\right)$$

$$m\frac{dv}{dt} + \frac{B^2l^2}{R}v = \frac{BlE}{R}$$

$$\frac{dv}{dt} + \frac{B^2l^2}{mR}v = \frac{BlE}{mR} \text{ dividing both sides by } m. \quad (36.6)$$

Solving the above equation with the boundary condition, at $t = 0$, $v = 0$, the expressions for velocity and current are obtained as follows:

$$v = \frac{E}{Bl}\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)$$

$$e = Blv = E\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)$$

$$i = \frac{E - e}{R} = \frac{E}{R}e^{-\frac{B^2l^2}{mR}}t \quad (36.7)$$

The figures 36.5 and 36.6 show the variation of current, velocity and emf induced in the conductor under motor mode condition.

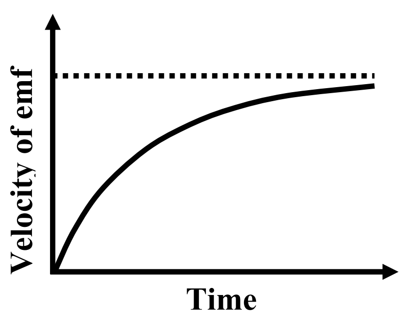


Figure 36.5: Variation of Velocity or emf.

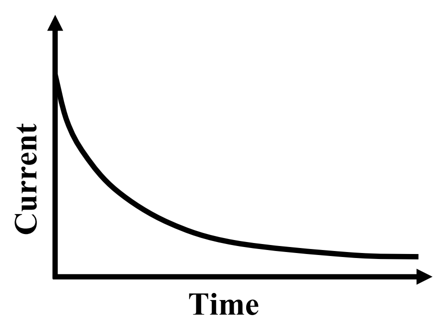


Figure 36.6: Variation of current.

The final current and speed in steady state will be 0 A and $v_0 = E/Bl$ m/sec. The current drawn from the supply becomes zero because battery voltage E is exactly balanced by the induced voltage in the conductor. This induced voltage in the conductor is called the back emf E_b. In this steady operating condition, note that $E_b = Blv_0 = E$. Both power drawn from the supply and mechanical output power will be zero. This is the no load steady operating point with $i = 0$ A and $v_0 = E/Bl$ m/sec.

Let us now investigate what will happen to the current and speed if a opposing force is present. Imagine the track ahead is not frictionless but offers a constant frictional force F_f. Let us start counting time afresh such that at $t = 0$ the conductor enters the frictional track. Due to inertia of the conductor speed can not change instantaneously i.e., at $t = 0^+$, $v = v_0$ which means $i\ (0^+)$ is also zero since $E_b\ (0^+) = E$. So at $t = 0^+$ no electromagnetic force acts on the conductor. Therefore velocity of the conductor must now start decreasing as F_f acts opposite to the direction of motion. But the moment v becomes less than v_0, E_b too will become less than E. Therefore the conductor will draw now current producing a driving force in the direction of motion. This dropping of speed and drawing more current from the battery will continue till the current magnitude reaches such a value which satisfies $F_e = BIl = F_f$. Obviously, the conductor draws a new steady state current of $I = F_f/Bl$. Similarly new steady speed can be calculated from the KVL equation

$$\begin{aligned} E_b + IR &= E \\ \text{or, } Blv + IR &= E \\ \text{or, } v &= (E - IR)/Bl \end{aligned}$$

One can of course write down a first order differential equation and solve it using the boundary condition mentioned above to know exactly how with time the initial current $i = 0$ changes to new steady state current I. Similarly expression for change of velocity with time, from initial value v_0 to the final steady value of $(E - IR)/Bl$ can easily be obtained.

It is however, **important to note** that one need not write and solve the differential equation to obtain steady state solution. Solution of algebraic equations, one involving force balance (driving force = opposing force) and the other involving voltage balance (KVL) in fact is sufficient.

36.3 Rotating Machines

In the previous section, we considered a single conductor moving with a velocity v along a straight track under the influence of a constant magnetic field throughout the track. This is certainly not a very attractive way of generating d.c voltage as the length of the generator becomes long(!) and one has to provide magnetic field all along the length. However, it brought out several important features regarding forces, current, back emf, steady and transient operation etc for both motor and generator operations.

Imagine that a straight conductor is glued on the surface of a cylinder structure parallel to the axial length of the cylinder. Let us also attach a shaft along the axis of the cylinder so that the structure is free to rotate about the shaft. In such a situation, the conductor will have rotational motion if the shaft is driven externally. Magnets (or electro magnets) can be fitted on a stationary structure producing radial magnetic fields as shown in the figure 36.7. The figure shows a sectional view with the shaft and the conductor perpendicular to the screen.

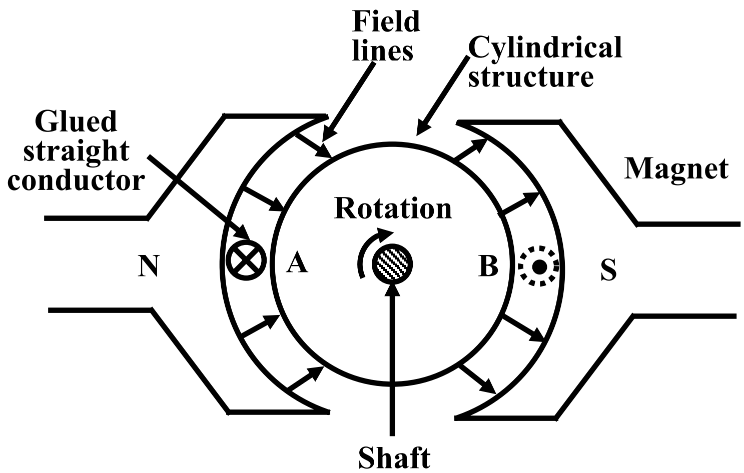


Figure 36.7: Single conductor rotating machine.

Thus, if the shaft is rotated at constant rpm, the conductor (glued to the surface of the cylinder) too will rotate at the same speed. In the process of rotation, the conductor will cut the lines of forces of magnetic field, in the same way as the straight conductor did in the linear version of the generator. Therefore, across the front end and the back end of the conductor, voltage will be induced. However, as the conductor moves, it some times cuts flux lines produced by N-pole and some other time it cuts flux lines produced by the south pole. The polarity of the induced voltage is therefore going to change. The sense of induced emf will be ⊗ when the conductor will be at position A (under the influence of north pole) and it will ⊙ when the conductor will be at position B (under the influence of the south pole). But we have already seen in the previous lesson, that this voltage can be made unidirectional by using commutator segments and brushes. The greatest advantage is that, the rotating machine now becomes of finite size and convenient for coupling prime mover or the mechanical load. Naturally, rotational speed and torques will be more useful quantities compared to velocity and forces in linear machines.

36.3.1 Driving & Opposing torques

There are various kinds of rotating electrical machines such as D.C machines, Induction machines, Synchronous machines etc. and they can run either as motor or as a generator. When a generator or a motor runs at a constant speed, we can say with conviction (from Newton's laws of rotational motion) that the *driving* torque and the *opposing* torque must be numerically equal and acting in opposite directions.

36.3.2 Generator mode

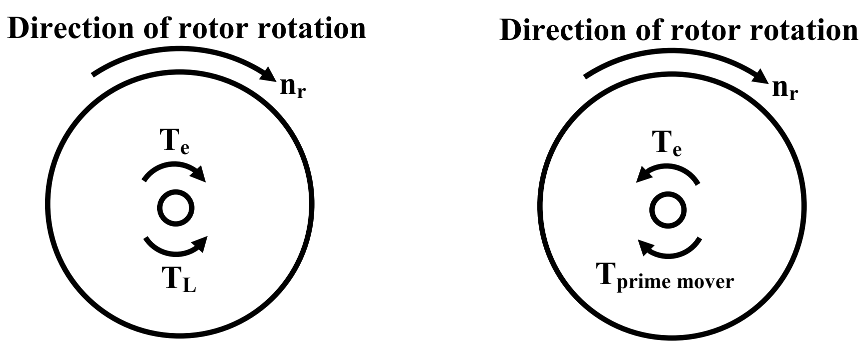
In case of generator mode, the driving torque is obtained by *prime movers.* A diesel engine or water turbine or steam turbine could be selected as prime movers. In laboratory environment, motors are used as prime movers. The direction of rotation of the generator is same as the direction of the prime mover torque. A loaded electrical rotating machine always produces electromagnetic torque T_e, due to the interaction of stator field and armature current. T_e together with small frictional torque is the opposing torque in generator mode. This opposing torque is called the *load* torque, T_L. If one wants to draw more electrical power out of the generator, T_e (hence T_L) increases due to more armature current. Therefore, prime mover torque must increase to balance T_L for steady speed operation with more fuel intake.

36.3.3 Motor mode

In case of motor mode, the driving torque is the electromagnetic torque, T_e and direction of rotation will be along the direction of T_e. Here the opposing torque will be due to mechanical load (such as pumps, lift, crane, blower etc.) put on the shaft and small frictional torque. In this case also the opposing torque is called the load torque T_L. For steady speed operation, $T_e = T_L$ numerically and acts in opposite direction. To summarize, remember:

- If it is acting as a motor, electromagnetic torque T_e acts along the direction of the rotor rotation and the load torque T_L acts in the opposite direction of rotation as shown in the figure 36.8 (a). If $T_e = T_L$ motor runs steadily at constant speed. During transient operation, if $T_e > T_L$, motor will accelerate and if $T_e < T_L$ motor will decelerate.

- On the other hand, if the machine is acting as a generator, the prime mover torque T_{pm} acts along the direction of rotation while the electromagnetic torque, T_e acts in the opposite direction of rotation as shown in figure 36.8 (b). Here also during transient operation if $T_{pm} > T_L$, the generator will accelerate and if $T_{pm} < T_L$, the generator will decelerate.

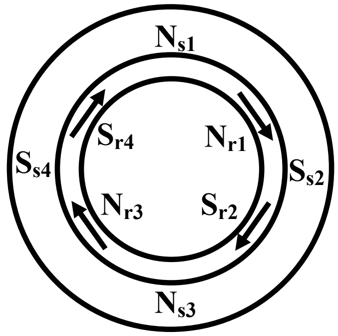


(a) Motor mode **(b) Generator mode**

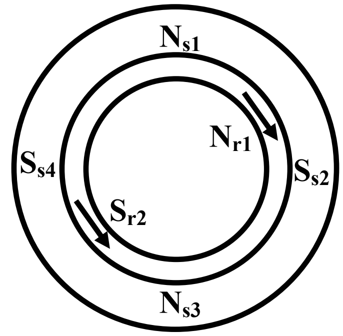
Figure 36.8: Direction of torques in rotating machines.

36.4 Condition for steady production of torque

The production of electromagnetic torque can be considered to be interaction between two sets of magnets, one produced due to current in the stator windings and the other produced due to rotor winding current. The rotor being free to rotate, it can only move along direction of the resultant torque. Let us first assume that windings are so wound that both stator and rotor produces same number of poles when they carry current. In figure 36.9(a) both stator and rotor produces 4 number of poles each. It can easily be seen that rotor pole N_{r1} is repelled by N_{s1} and attracted by S_{s2}. These two forces being additive produces torque in the clock wise direction. In the same way other rotor poles experience torque along the same clock wise direction confirming that a resultant torque is produced.



(a) Torques are additive when no. of Stator poles = Rotor poles



(b) Rotor experiences zero torque when no. of Stator poles ≠ Rotor poles

Figure 36.9: Direction of torques in rotating machines.

However, if stator winding produces 4 poles and rotor winding produces 2 poles, the resultant torque experienced by rotor will be zero as shown in figure 36.9(b). Here N_{r1} is repelled by N_{s1} and attracted by S_{s2} trying to produce torque in the clock wise direction: while S_{r2} is attracted by N_{s3} and repelled by S_{s4} trying to produce torque in the counter clock wise direction. So net torque is zero. So a rotating electrical machine can not work with different number of poles.

The condition that stator number of poles should be equal to the rotor number of poles is actually a necessary condition for production of steady electromagnetic torque. What is the sufficient condition then? Let us look once again at figure 36.9(a) where stator and rotor number of poles are same and equal to 4. Suppose the relative position of the poles shown, is at a particular instant of time say *t*. We can easily recognize the factors on which the magnitude of the torque produced will depend at this instant. Strength of stator & rotor poles is definitely one factor and the other factor is the relative position of stator and rotor poles (which essentially means the distance between the interacting poles). If the machine has to produce a definite amount of torque for all time to come for sustained operation, the relative position of the stator and rotor field patterns must remain same and should not alter with time. Alternative way of expressing this is to say : *the relative speed between the stator and rotor fields should be zero with respect to a stationary observer.*

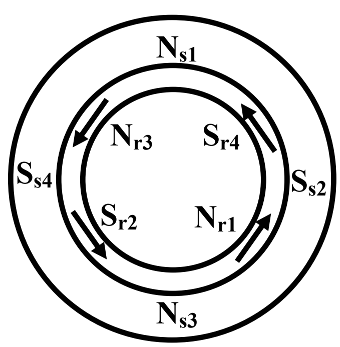


Figure 36.10: Direction of torques in rotating machines.

Now let us see what happens if there exists a relative speed between the stator and the rotor fields. Suppose initial positions of the two fields are as shown in figure 36.9(a), when the direction of the torque is clock wise. Due to the relative speed, let after some time the position of the field patterns becomes as shown in figure 36.10. At this instant, we see that direction of torque reverses and becomes counter clock wise. In other words rotor will go on experiencing alternating torque, sometimes in the clock wise and sometimes in counter clock wise direction. Hence net average torque (over time), will be zero. Summarizing the above we can conclude that a rotating electrical machine can produce steady electromagnetic torque only when the following two conditions are satisfied.

- Stator and rotor number of poles must be same.
- There should not be a relative speed existing between the two fields with respect to a stationary observer.

36.5 D.C generator: Basic principle of operation

A D.C generator is shown in figure 36.11. Let the armature be driven by a prime mover in the clock wise direction and the stator field is excited so as to produce the stator poles as shown. There will be induced voltage in each armature conductor. The direction of the induced voltage can be ascertained by applying *Fleming's right hand rule.* All the conductors under the influence of south pole will have $\otimes$ directed induced voltage, while the conductors under the influence of North pole will have $\odot$ induced voltage in them. For a loaded generator the direction of the armature current will be same as that of the induced voltages. Thus $\otimes$ and $\odot$ also represent the direction of the currents in the conductors. We know, a current carrying conductor placed in a magnetic field experiences force, the direction of which can be obtained by applying *Fleming's left hand rule.* Applying this rule to the armature conductors in figure 36.11, we note that rotor experiences a torque (T_e) in the counter clockwise direction (i.e., opposite to the direction of rotation). As discussed earlier, for steady speed operation $T_{pm} = T_e$.

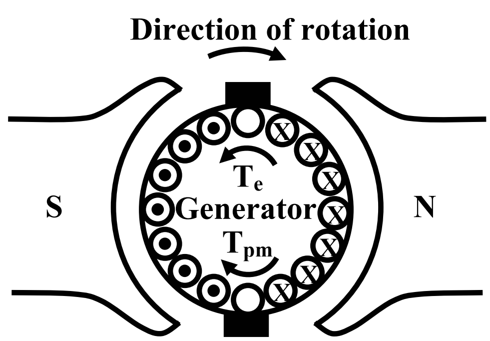


Figure 36.11: D.C Generator: principle of operation.

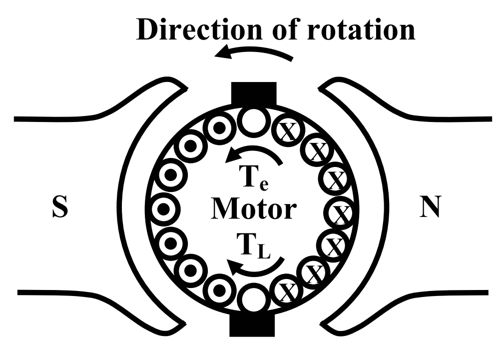


Figure 36.12: D.C motor: principle of operation.

36.6 D.C motor: Basic principle of operation

Now let us look at a D.C motor shown in figure 36.12. Excited stator coils are assumed to produce south and north poles as depicted. Now if the armature is connected to a D.C source, current will be flowing through the armature conductors. Let the conductors under the influence of the south pole carries $\otimes$ currents and the conductors under the influence of north pole carries $\odot$ currents. Applying *Fleming's left hand rule,* we note torque T_e will be produced in the counter clockwise direction causing the rotor to move in the same direction. For steady speed operation, T_e will be balanced by the mechanical load torque T_L imposed on the shaft. Load torque T_L will be in the opposite direction of rotation. It should be noted that when the armature conductors move in presence of a field, voltage is bound to be induced in the conductor (as explained in the previous section). The direction of this generated emf, ascertained by *Fleming's right hand rule* is found to be in the opposite direction of the current flow. In other words, the generated voltage in the armature acts in opposition to the source voltage. The generated voltage in a D.C motor is usually called the *back emf, E_b.* The expression for armature current is $I_a = \frac{(V_s - E_b)}{r_a}$, where V_s is the supply voltage and r_a is the armature circuit resistance.

36.7 Answer the following

1. Show the directions of prime mover torque, electromagnetic torque and direction of rotation for an electrical generator.
2. Show the directions of electromagnetic torque, load torque and direction of rotation of an electrical motor.
3. Write down the conditions for production of steady electromagnetic torque for a rotating electrical motor.
4. The operation of electrical rotating machines can be thought to be the interaction of two sets of magnet, one produced by stator coils and the other produced by rotor coils. If stator produces 6 poles, explain how many poles rotor must produces for successful steady operation.
5. A single conductor motor as shown in figure 36.4 is found to draw a steady 0.5 A current from d,c supply of E = 50 V. If l = 2 m and B = 1.2 T, Calculate (i) the back emf E_b, (ii) the velocity of the conductor, (iii) the driving force and (iv) the opposing force. Also check the for the power balance in the system i.e., power supplied by the battery must be equal to the power loss in the resistance + mechanical power to overcome friction.